Boosting thermalization of classical and quantum many-body systems

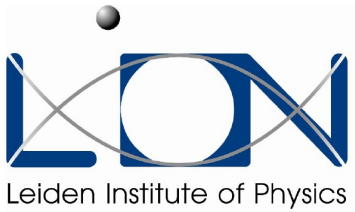
Jinfu Chen | @Qtech'25-budapest, 24.9.2025



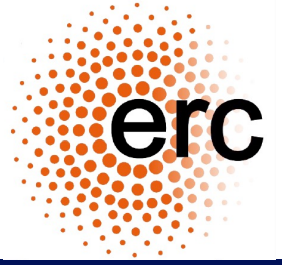
Universiteit Leiden
The Netherlands

arXiv:2411.03420 & ongoing work with **Kshiti Sneh Rai, Patrick Emonts, Donato Farina, Marcin Płodzień, Przemyslaw Grzybowski, Maciej Lewenstein, Jordi Tura**









Thermal state preparation

Classical Markov processes
Master equation

$$\frac{d}{dt}|p\rangle = \boldsymbol{L}\,|p\rangle$$

Quantum Gibbs Sampling



Open quantum systems
Lindbladian

$$\frac{d}{dt}\boldsymbol{\rho} = \mathcal{L}[\boldsymbol{\rho}]$$

- Prepare classical & quantum thermal states
  - **Design kinetic rules** $\boldsymbol{L}$ or $\mathcal{L}$ → Unique steady state
  - **Optimize kinetic coefficients** → Fast thermalization

C.-F. Chen, M. J. Kastoryano, A. Gilyén, An efficient and exact noncommutative quantum Gibbs sampler (2023)
A. Gilyén, C.-F. Chen, J. F. Doriguello, M. J. Kastoryano, Quantum generalizations of Glauber and Metropolis dynamics (2024)

Preparing classical thermal states

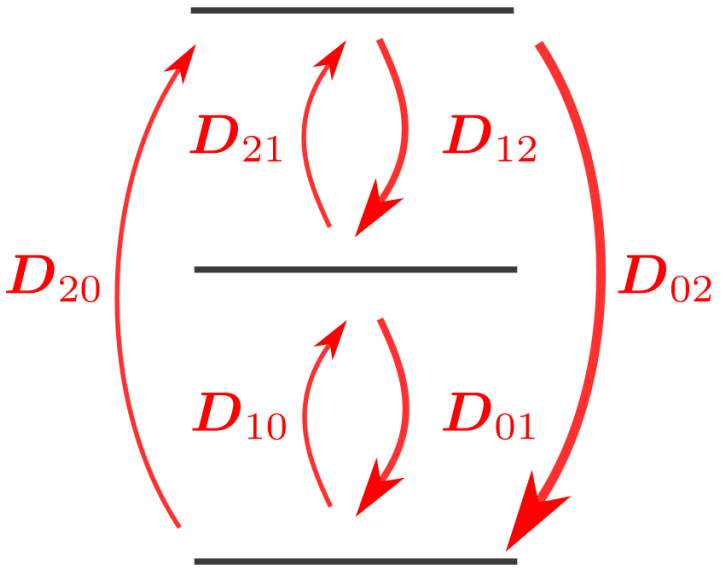
- Three-level system
- Detailed-balanced condition: $\dfrac{D_{ij}}{D_{ji}} = e^{\beta(E_j - E_i)}$ ➡ $D^\top = \rho_\beta^{-1} D \rho_\beta$
- Liouvillian

$$\rho_\beta = e^{-\beta h^{\mathrm{cl}}}/Z(\beta)$$

dissipation jump

$$D = \begin{pmatrix} D_{00} & \tilde{\gamma}_{01} e^{\beta(E_1-E_0)/2} & \tilde{\gamma}_{02} e^{\beta(E_2-E_0)/2} \\ \tilde{\gamma}_{01} e^{-\beta(E_1-E_0)/2} & D_{11} & \tilde{\gamma}_{12} e^{\beta(E_2-E_1)/2} \\ \tilde{\gamma}_{02} e^{-\beta(E_2-E_0)/2} & \tilde{\gamma}_{12} e^{-\beta(E_2-E_1)/2} & D_{22} \end{pmatrix}$$

$$D_{00} = -\tilde{\gamma}_{01} e^{-\beta(E_1-E_0)/2} - \tilde{\gamma}_{02} e^{-\beta(E_2-E_0)/2}$$
$$D_{11} = -\tilde{\gamma}_{01} e^{\beta(E_1-E_0)/2} - \tilde{\gamma}_{12} e^{-\beta(E_2-E_1)/2}$$
$$D_{22} = -\tilde{\gamma}_{02} e^{\beta(E_2-E_0)/2} - \tilde{\gamma}_{12} e^{\beta(E_2-E_1)/2}$$



- Three kinetic coefficients $\tilde{\gamma}_{ij}$ can be optimized to boost thermalization. $|\rho_{\mathrm{eq}}\rangle = \rho_\beta |\mathbb{1}\rangle$

Motivation: boosting thermalization

- Thermalization dynamics

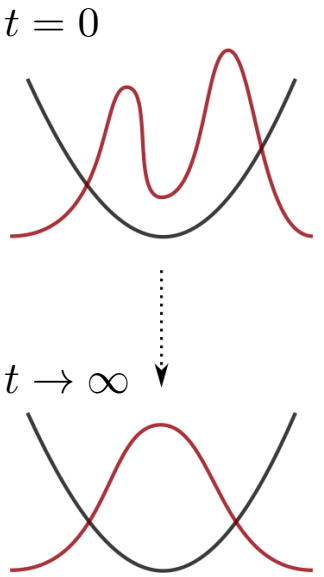
$$\dot{\boldsymbol{\rho}} = \mathcal{L}[\boldsymbol{\rho}]$$

Slowest relaxing mode

$$\boldsymbol{\rho}(t) = \boldsymbol{\rho}_\beta + \boldsymbol{\rho}_1 e^{-\lambda_1 t} + \boldsymbol{\rho}_2 e^{-\lambda_2 t} + \cdots$$

$$\lambda_0 = 0$$

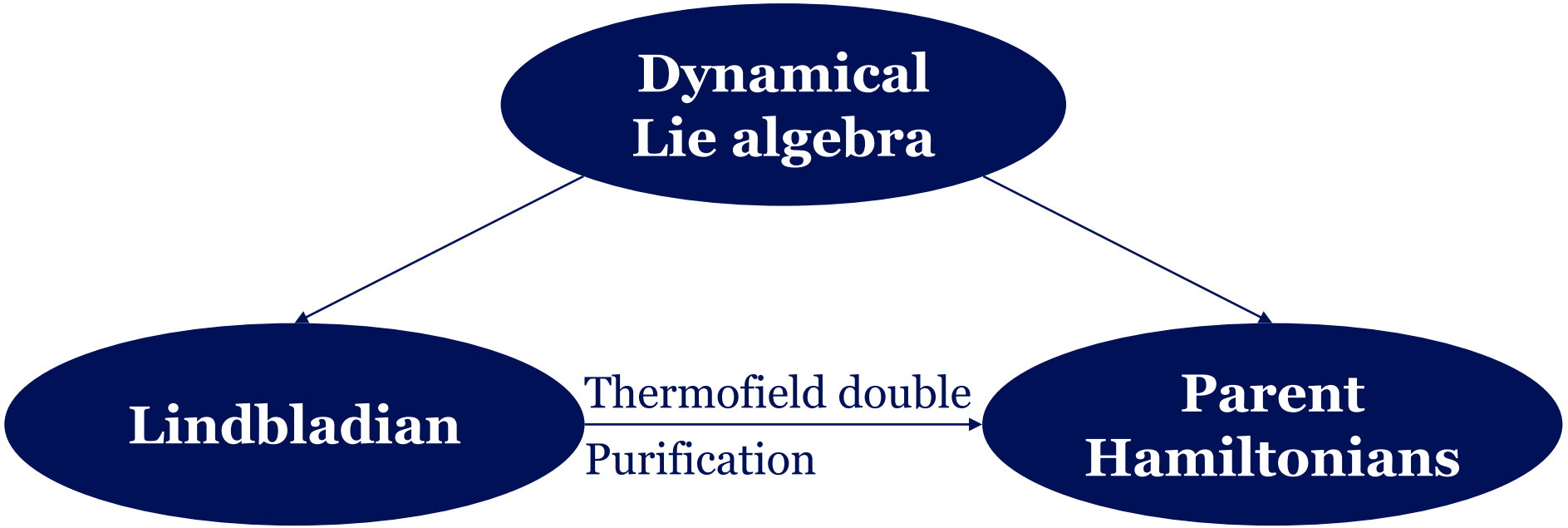
Spectral gap $\quad \mathrm{Re}(\lambda_1) \leq \mathrm{Re}(\lambda_2) \leq \cdots$



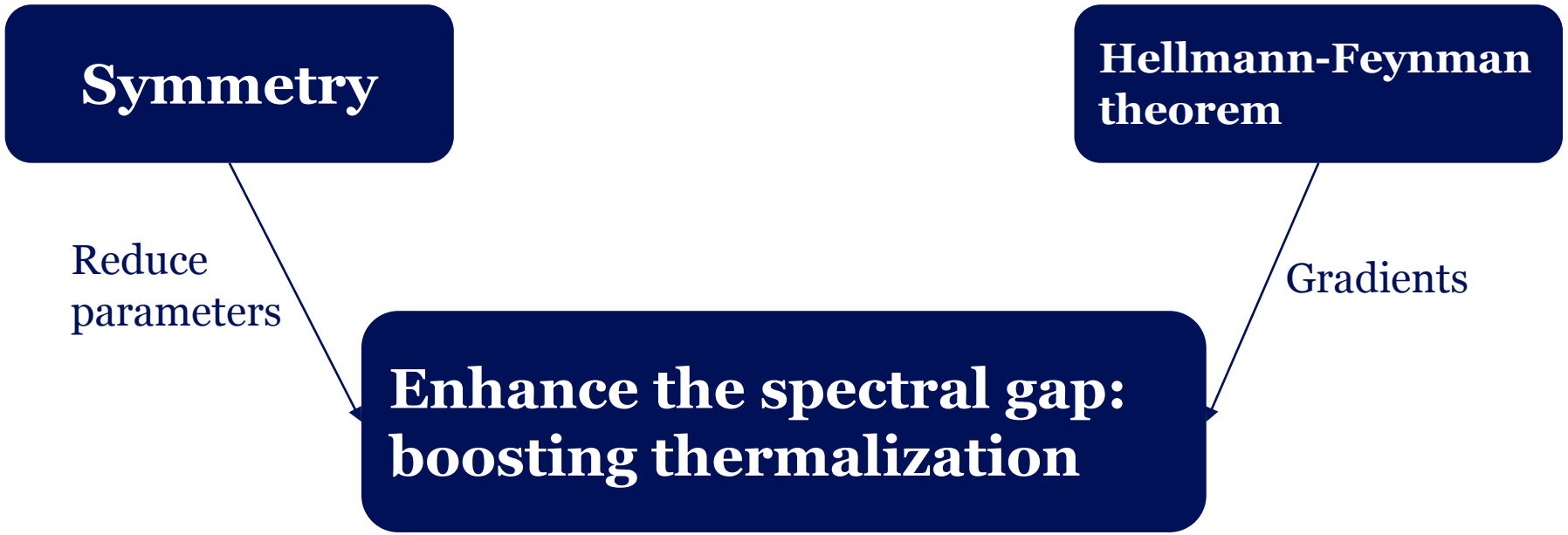
Maximizing $\mathrm{Re}(\lambda_1)$ ➡ Fast thermalization ➡ Reducing irreversibility

- For many-body systems, $\mathcal{L}$ has too many free parameters to determine.

Construction of Lindbladians



Optimization of spectral gap



Content

- Preparing classical thermal states
  - Kinetic Ising model
  - Spectral gap optimization
- Preparing quantum thermal states
  - Constructing Lindbladians under quantum detailed balance
  - Dynamical Lie algebra approach
  - Lipkin-Meshkov-Glick model
  - Transverse Ising model
- Summary

Infinite-temperature Liouvillians

- Kinetic Ising model: classical spin dynamics under stochastic flips
- We first construct Liouvillians to prepare infinite-temperature state

Liouvillians

$$\tilde{D} = \sum_i \tilde{\gamma}_i (\tilde{A}_i - \tilde{A}_i^\top \tilde{A}_i)$$

Detailed balance condition

$$\tilde{D}^\top = \tilde{D}$$

$$\sum_i \tilde{\gamma}_i \tilde{A}_i = \sum_i \tilde{\gamma}_i \tilde{A}_i^\top$$

- Single spin flip operator: $\tilde{A}_i = \sigma_i^x$

$$\tilde{D} = -\tilde{H} = \sum_{i=1}^{N} \tilde{\gamma}_i (\sigma_i^x - I)$$

Finite-temperature Liouvillians

- Finite-temperature thermal state

Liouvillians

$$D = \sum_i \tilde{\gamma}_i (\rho_\beta^{1/2} \tilde{A}_i \rho_\beta^{-1/2} - A_i^\top A_i)$$

$$A_i = \rho_\beta^{1/4} \tilde{A}_i \rho_\beta^{-1/4}$$

Detailed balance condition

$$D^\top = \rho_\beta^{-1} D \rho_\beta$$

- Ising Hamiltonian $h^{\mathrm{cl}} = \sum_{i=1}^{N} -J \sigma_i^z \sigma_{i+1}^z$

- Flip operator $A_i = \cosh\left(\frac{\beta J}{2}\right)^2 \sigma_i^x - \frac{i}{2}\sinh(\beta J)(\sigma_{i-1}^z + \sigma_{i+1}^z)\sigma_i^y + \sinh\left(\frac{\beta J}{2}\right)^2 \sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^z$

Parent Hamiltonians

Parent Hamiltonians

$$\boldsymbol{H} = -\boldsymbol{\rho}_\beta^{-1/2} \boldsymbol{D} \boldsymbol{\rho}_\beta^{1/2}$$
$$= \sum_i \tilde{\gamma}_i \boldsymbol{H}_i = \sum_i \tilde{\gamma}_i (\boldsymbol{A}_i^\top \boldsymbol{A}_i - \tilde{\boldsymbol{A}}_i)$$

- Parent Hamiltonian of kinetic Ising model

$$\boldsymbol{H}_i = \frac{1 + \eta \boldsymbol{\sigma}_i^z (\boldsymbol{\sigma}_{i-1}^z + \boldsymbol{\sigma}_{i+1}^z) + \eta^2 \boldsymbol{\sigma}_{i-1}^z \boldsymbol{\sigma}_{i+1}^z}{1 - \eta^2} - \boldsymbol{\sigma}_i^x \qquad \eta = \tanh(\beta J)$$

- Other choices of single spin flip operators

$$\tilde{\boldsymbol{A}}_{in} = (1 \pm \boldsymbol{\sigma}_{i-1}^z \boldsymbol{\sigma}_{i+1}^z) \boldsymbol{\sigma}_i^x \qquad \boldsymbol{H} = \sum_{i=1}^N \tilde{\gamma}_i (1 + \delta_i \boldsymbol{\sigma}_{i-1}^z \boldsymbol{\sigma}_{i+1}^z) \boldsymbol{H}_i$$

Liouvillians and Parent Hamiltonians

	Liouvillians $D = \sum_i \tilde{\gamma}_i D_i$	**Parent Hamiltonians** $H = \sum_i \tilde{\gamma}_i H_i$
Steady state/ ground state	$\lvert p_\beta \rangle = \frac{e^{-\beta \boldsymbol{h}^{\mathrm{cl}}}}{Z(\beta)} \lvert \mathbb{1} \rangle$ $\boldsymbol{D} \lvert p_\beta \rangle = 0$ $\langle \mathbb{1} \rvert \boldsymbol{D} = 0$	$\lvert \sqrt{p_\beta} \rangle = \frac{e^{-\beta \boldsymbol{h}^{\mathrm{cl}}/2}}{\sqrt{Z(\beta)}} \lvert \mathbb{1} \rangle$ $\boldsymbol{H} \lvert \sqrt{p_\beta} \rangle = 0$
Detailed balance condition	$\boldsymbol{\rho}_\beta^{-1} \boldsymbol{D} \boldsymbol{\rho}_\beta = \boldsymbol{D}^\top$	$\boldsymbol{H}_i^\dagger = \boldsymbol{H}_i^T = \boldsymbol{H}_i$
Spectrum	$-\lambda_0, -\lambda_1, \ldots, -\lambda_{d-1}$	$\lambda_0, \lambda_1, \ldots, \lambda_{d-1}$

$$\lambda_0 = 0, \ \lambda_1 > 0$$

Spectral gap optimization

- Boosting thermalization: optimize $\tilde{\gamma}_i$ to enhance the spectral gap λ_1

- First excited state: $|\Phi_1\rangle$, **Hellmann–Feynman theorem** → derivatives

$$\frac{\partial \lambda_1}{\partial \tilde{\gamma}_i} = \langle \Phi_1 | \frac{\partial \mathcal{H}}{\partial \tilde{\gamma}_i} | \Phi_1 \rangle$$

- λ_1 is extensive with $\tilde{\gamma}$.

Normalized gap

$$\Delta = \frac{\lambda_1}{\mathrm{tr}(\mathcal{D})/d^2} = \frac{\lambda_1}{\sum_{j=1}^{d^2-1} \lambda_j / d^2}$$

Symmetry

G_Φ: symmetry group of $|\Phi_0\rangle\langle\Phi_0|$
G_H: symmetry group of H

Maximize the gap Δ ➡ $G_H = G_\Phi$

- **Concavity:** $G_H \geq G_\Phi$

Reduce many free parameters in $\tilde{\gamma}_i$

The gap increases when H obeys the symmetry of $|\Phi_0\rangle$.

- **Spontaneous symmetry breaking:** $G_H \leq G_\Phi$

Otherwise, $U_{g_H}|\Phi_0\rangle$ is another ground state.

Kshiti Sneh Rai, Jin-Fu Chen, Patrick Emonts, Jordi Tura, Spectral Gap Optimization for Enhanced Adiabatic State Preparation, Phys. Rev. Lett. 135, 090602 (2025)

Application: Kinetic Curie-Weiss model

- Classical Hamiltonian

$$\boldsymbol{h}^{\mathrm{cl}} = -\frac{1}{N} J \sum_{i<j} \boldsymbol{\sigma}_i^z \boldsymbol{\sigma}_j^z.$$

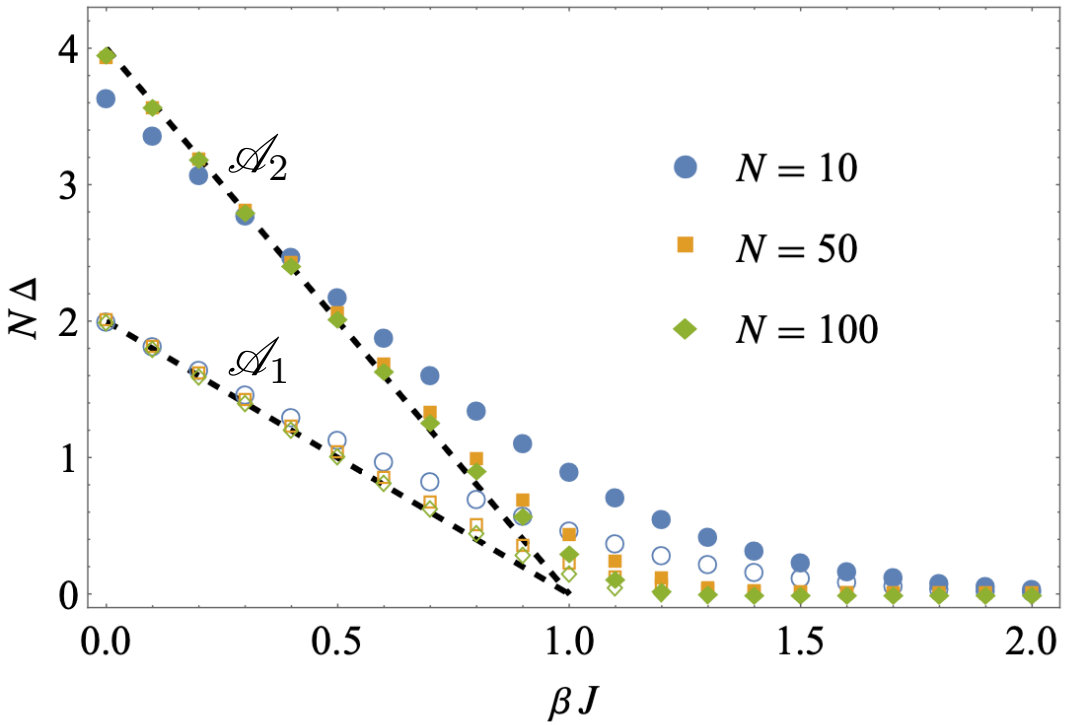
- Single spin flip:

$$\tilde{\mathscr{A}}_1 = \{\boldsymbol{\sigma}_i^x | 1 \leq i \leq N\}$$

- Double spin flip:

$$\tilde{\mathscr{A}}_2 = \{\boldsymbol{\sigma}_i^x, \boldsymbol{\sigma}_i^x \boldsymbol{\sigma}_j^x | 1 \leq i < j \leq N\}$$

Permutation symmetry
Local Z2 symmetry



2-nd phase transition at $\beta J = 1$
Spectral gap closes for large N (ergodicity breaking)

Application: 1D Kinetic Ising model model

- Commuting local terms

$$\boldsymbol{h}^{\mathrm{cl}} = -\sum_{i=1}^{N} J\boldsymbol{\sigma}_i^z\boldsymbol{\sigma}_{i+1}^z.$$

- Single spin flip

$$\tilde{\mathscr{A}}_1 = \{\boldsymbol{\sigma}_i^x | 1 \le i \le N\}$$

$$\tilde{\mathscr{A}}_2 = \{\tilde{\boldsymbol{A}}_{i1}, \tilde{\boldsymbol{A}}_{i2} | 1 \le i \le N\}$$

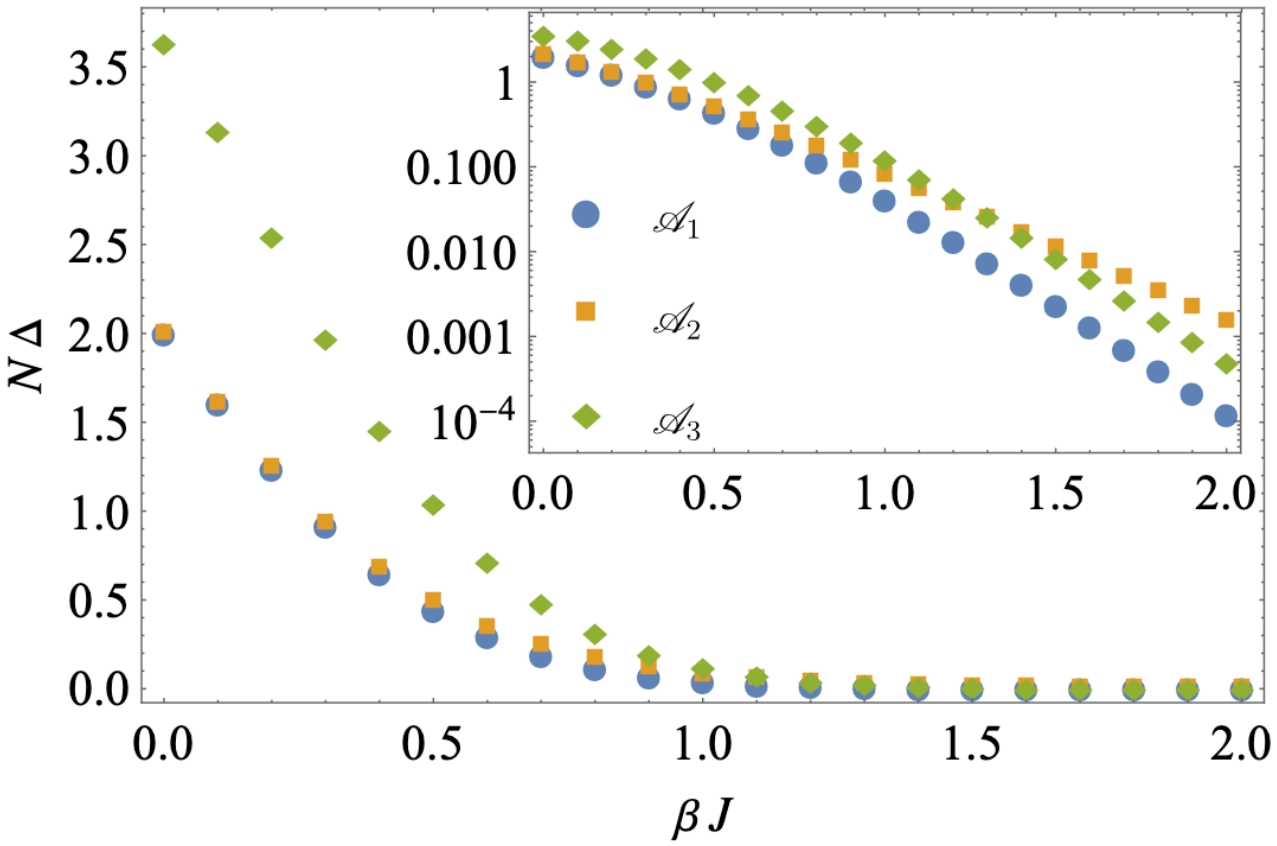
$$\tilde{\boldsymbol{A}}_{i12} = (1 \pm \boldsymbol{\sigma}_{i-1}^z\boldsymbol{\sigma}_{i+1}^z)\boldsymbol{\sigma}_i^x/2$$

- Double spin flip

$$\tilde{\mathscr{A}}_3 = \{\boldsymbol{\sigma}_i^x, \boldsymbol{\sigma}_i^x\boldsymbol{\sigma}_{i+1}^x | 1 \le i \le N\}$$

Translational symmetry
Local Z2 symmetry

$N = 10$



Preparing quantum thermal states

- **We apply the spectral gap optimization to quantum Gibbs sampling**

- Lindbladian and adjoint Lindbladian

$$\mathcal{L}[\bullet] = -i[\boldsymbol{K}, \bullet] - \frac{1}{2}\{\boldsymbol{R}, \bullet\} + \sum_i \gamma_i \boldsymbol{A}_i \bullet \boldsymbol{A}_i^\dagger$$

$$\mathcal{L}^\sharp[\bullet] = i[\boldsymbol{K}, \bullet] - \frac{1}{2}\{\boldsymbol{R}, \bullet\} + \sum_i \gamma_i \boldsymbol{A}_i^\dagger \bullet \boldsymbol{A}_i$$

unitary dissipation jump

$$\boldsymbol{R} = \sum_i \gamma_i \boldsymbol{A}_i^\dagger \boldsymbol{A}_i$$

Quantum detailed balance condition

- Existing some Hermitian $\boldsymbol{Q}$, such that

$$\mathcal{L}^{\sharp}[\bullet] - \boldsymbol{\rho}_{\beta}^{-1/2}\mathcal{L}[\boldsymbol{\rho}_{\beta}^{1/2}\bullet\boldsymbol{\rho}_{\beta}^{1/2}]\boldsymbol{\rho}_{\beta}^{-1/2} = 2i[\boldsymbol{Q},\bullet],$$

$$\boldsymbol{\rho}_{\beta} = e^{-\beta \boldsymbol{h}}/Z(\beta)$$

$$[\boldsymbol{Q},\boldsymbol{\rho}_{\beta}] = 0,$$

- Decompose it into

$$\mathcal{L}[\bullet] = \mathcal{Q}[\bullet] + \mathcal{D}[\bullet]$$

$$\mathcal{Q}[\bullet] = -i[\boldsymbol{Q},\bullet] = -\mathcal{Q}^{\sharp}[\bullet]$$

$$\mathcal{D}^{\sharp}[\bullet] = \boldsymbol{\rho}_{\beta}^{-1/2}\mathcal{D}[\boldsymbol{\rho}_{\beta}^{1/2}\bullet\boldsymbol{\rho}_{\beta}^{1/2}]\boldsymbol{\rho}_{\beta}^{-1/2}$$

F. Fagnola, V. Umanità, *Commun. Math. Phys.* **298**, 523–547 (2010).

Infinite-temperature Lindbladians

Lindbladians

$$\tilde{\mathcal{D}}[\bullet] = \sum_i \tilde{\gamma}_i (\tilde{A}_i \bullet \tilde{A}_i^\dagger - \tfrac{1}{2}\{\tilde{A}_i^\dagger \tilde{A}_i, \bullet\})$$

Detailed balance condition

$$\tilde{\mathcal{D}}^\sharp[\bullet] = \tilde{\mathcal{D}}[\bullet]$$

$$\sum_i \tilde{\gamma}_i \tilde{A}_i \bullet \tilde{A}_i^\dagger = \sum_i \tilde{\gamma}_i \tilde{A}_i^\dagger \bullet \tilde{A}_i$$

Pairs of Lindblad operators:

$$\tilde{A}_{n'} = \tilde{A}_n^\dagger \in \tilde{\mathscr{A}}$$

$$\tilde{\gamma}_{n'} = \tilde{\gamma}_n$$

Finite-temperature Lindbladian

Lindbladians and detailed balance condition

$$\mathcal{D}[\bullet] = -i[\boldsymbol{K}, \bullet] + \sum_i \tilde{\gamma}_n (\boldsymbol{A}_i \bullet \boldsymbol{A}_i^\dagger - \tfrac{1}{2}\{\boldsymbol{A}_i^\dagger \boldsymbol{A}_i, \bullet\})$$

$$\mathcal{D}^\sharp[\bullet] = \boldsymbol{\rho}_\beta^{-1/2} \mathcal{D}[\boldsymbol{\rho}_\beta^{1/2} \bullet \boldsymbol{\rho}_\beta^{1/2}] \boldsymbol{\rho}_\beta^{-1/2}$$

Lindblad operators and unitary term

$$\boldsymbol{A}_i = \boldsymbol{\rho}_\beta^{1/4} \tilde{\boldsymbol{A}}_i \boldsymbol{\rho}_\beta^{-1/4}$$

$$\{\boldsymbol{\rho}_\beta^{-\frac{1}{2}}, \boldsymbol{\rho}_\beta^{\frac{1}{4}} \boldsymbol{K} \boldsymbol{\rho}_\beta^{\frac{1}{4}}\} = \frac{i}{2}[\boldsymbol{\rho}_\beta^{-\frac{1}{2}}, \boldsymbol{\rho}_\beta^{\frac{1}{4}} \boldsymbol{R} \boldsymbol{\rho}_\beta^{\frac{1}{4}}]$$

- Locality of $\boldsymbol{A}_i$ can be improved by choosing $\widetilde{\boldsymbol{A}}_i$ with filtering.

Parent Hamiltonian

$$\mathcal{H}[\cdot] \equiv -\rho_\beta^{-1/4}\mathcal{D}[\rho_\beta^{1/4}\cdot\rho_\beta^{1/4}]\rho_\beta^{-1/4}.$$

$$\mathcal{D}^\sharp[\cdot] = \rho_\beta^{-1/2}\mathcal{D}[\rho_\beta^{1/2}\cdot\rho_\beta^{1/2}]\rho_\beta^{-1/2}$$

$$\mathcal{H}[\bullet] = \mathcal{H}^\sharp[\bullet]$$

Parent Hamiltonian

$$\mathcal{H}[\bullet] = \tfrac{1}{2}\{\boldsymbol{N}, \bullet\} - \textstyle\sum_i \tilde{\gamma}_i \tilde{\boldsymbol{A}}_i \bullet \tilde{\boldsymbol{A}}_i^\dagger$$

$$\boldsymbol{\rho}_\beta^{-1/4}\boldsymbol{N}\boldsymbol{\rho}_\beta^{1/4} + \boldsymbol{\rho}_\beta^{1/4}\boldsymbol{N}\boldsymbol{\rho}_\beta^{-1/4} = 2\textstyle\sum_i \tilde{\gamma}_i \boldsymbol{A}_i^\dagger \boldsymbol{A}_i$$

Dynamical Lie algebra approach

- Heisenberg evolution of Lindblad operator:

$$\tilde{\boldsymbol{A}}_i(t) = e^{i\boldsymbol{h}t}\tilde{\boldsymbol{A}}_i e^{-i\boldsymbol{h}t} = \mathfrak{a}_i^\dagger e^{L_{\mathfrak{a}_i}t}E_0$$

$$\mathfrak{a}_i^\dagger = (\mathfrak{a}_i^{(0)}, \mathfrak{a}_i^{(1)}, \ldots, \mathfrak{a}_i^{(l)}, \ldots) \qquad \mathfrak{a}_i^{(0)} = \tilde{\boldsymbol{A}}_i$$

$$E_0 = (1, 0, 0, \ldots, 0)^\top$$

- Krylov expansion: $$i[\boldsymbol{h}, \mathfrak{a}_i^{(l)}] = \mathfrak{a}_i^{(l-1)} L_{\mathfrak{a}_i}^{(l-1,l)} + \mathfrak{a}_i^{(l+1)} L_{\mathfrak{a}_i}^{(l+1,l)}$$

$$\boldsymbol{A}_i = \tilde{\boldsymbol{A}}_i(\frac{i\beta}{4}) = \mathfrak{a}_i^\dagger e^{iL_{\mathfrak{a}_i}\beta/4}E_0$$

Dynamical Lie algebra approach

$$\boldsymbol{R} = \sum_i \tilde{\gamma}_i \boldsymbol{A}_i^\dagger \boldsymbol{A}_i = \sum_i \tilde{\gamma}_i \mathfrak{a}_i^\dagger e^{-i\beta L_{\mathfrak{a}_i}/4} E_{00} e^{-i\beta L_{\mathfrak{a}_i}/4} \mathfrak{a}_i$$
$$= r_0 \boldsymbol{I} + r_1 \hat{\boldsymbol{R}}$$

- Heisenberg evolution of the dissipation term: $\hat{\boldsymbol{R}}(t) = e^{i\boldsymbol{h}t} \hat{\boldsymbol{R}} e^{-i\boldsymbol{h}t} = \mathfrak{r}^\dagger e^{L_\mathfrak{r} t} E_0$

  $E_0 = (1, 0, 0, \ldots, 0)^\top$

- Energy domain

$$\boldsymbol{K} = \sum_\nu \frac{i}{2} \tanh\left(\frac{\beta\nu}{4}\right) \boldsymbol{R}_\nu \qquad \boldsymbol{N} = \sum_\nu \frac{1}{\cosh(\beta\nu/4)} \boldsymbol{R}_\nu$$

- Time domain

$$\boldsymbol{K} = \text{p.v.} \int_{-\infty}^{\infty} \frac{1}{\beta \sinh\left(\frac{2\pi t}{\beta}\right)} \boldsymbol{R}(t) dt \qquad \boldsymbol{N} = \int_{-\infty}^{\infty} \frac{2\boldsymbol{R}(t)}{\beta \cosh\left(\frac{2\pi t}{\beta}\right)} dt$$

- Dynamical Lie algebra

$$\boldsymbol{K} = r_1 \mathfrak{r}^\dagger \frac{\beta}{2} \tan\left(\frac{\beta}{4} L_\mathfrak{r}\right) E_0 \qquad \boldsymbol{N} = r_0 + r_1 \mathfrak{r}^\dagger \sec\left(\frac{\beta L_\mathfrak{r}}{4}\right) E_0$$

CKG algorithm

- Dynamical Lie algebra result for the CKG algorithm: C.-F. Chen, M. J. Kastoryano, A. Gilyén, (2023)
- The operator Fourier transform can be represented as

$$\hat{\boldsymbol{B}}_i(\tilde{\omega}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\tilde{\omega}t} f(t) \boldsymbol{B}_i(t) dt \qquad f(t) = e^{-\sigma_E^2 t^2} \sqrt{\sigma_E \sqrt{2/\pi}}$$

$$= \frac{e^{-\tilde{\omega}^2/(4\sigma_E^2)}}{(2\pi\sigma_E^2)^{1/4}} \mathfrak{b}_i^\dagger e^{\frac{L_{\mathfrak{b}_i}^2 - 2i\tilde{\omega} L_{\mathfrak{b}_i}}{4\sigma_E^2}} E_0, \qquad \boldsymbol{B}_i(t) = \mathfrak{b}_i^\dagger e^{L_{\mathfrak{b}_i} t} E_0.$$

- For $\boldsymbol{K}$ and $\boldsymbol{N}$, the construction from the dynamical Lie algebra is similar.

Quantum vs. Classical

Quantum detailed balance condition

$$\mathcal{D}^{\sharp}[\bullet] = \boldsymbol{\rho}_{\beta}^{-1/2}\mathcal{D}[\boldsymbol{\rho}_{\beta}^{1/2} \bullet \boldsymbol{\rho}_{\beta}^{1/2}]\boldsymbol{\rho}_{\beta}^{-1/2}$$

Parent Hamiltonian in
extended Hilbert space

$$\mathcal{H}[\bullet] = -\boldsymbol{\rho}_{\beta}^{-1/4}\mathcal{D}[\boldsymbol{\rho}_{\beta}^{1/4} \bullet \boldsymbol{\rho}_{\beta}^{1/4}]\boldsymbol{\rho}_{\beta}^{-1/4}$$

e.g., Lipkin-Meshkov-Glick model,
Transverse-Field Ising model

Detailed balanced condition

$$\boldsymbol{D}^{\top} = \boldsymbol{\rho}_{\beta}^{-1}\boldsymbol{D}\boldsymbol{\rho}_{\beta}$$

Parent Hamiltonian

$$\boldsymbol{H} = -\boldsymbol{\rho}_{\beta}^{-1/2}\boldsymbol{D}\boldsymbol{\rho}_{\beta}^{1/2}$$

e.g., Kinetic Ising model

Lipkin-Meshkov-Glick model

$$\tilde{\beta}J = N\beta J = 5$$

$$\tilde{\Delta} = \Delta N(N+2)/4$$

- LMG Hamiltonian

$$\boldsymbol{h} = -J[\frac{1}{N}\sum_{i<j}(\boldsymbol{\sigma}_i^z\boldsymbol{\sigma}_j^z + r\sum_{i<j}\boldsymbol{\sigma}_i^y\boldsymbol{\sigma}_j^y) + g\sum_i \boldsymbol{\sigma}_i^x]$$

- In the symmetric subspace,

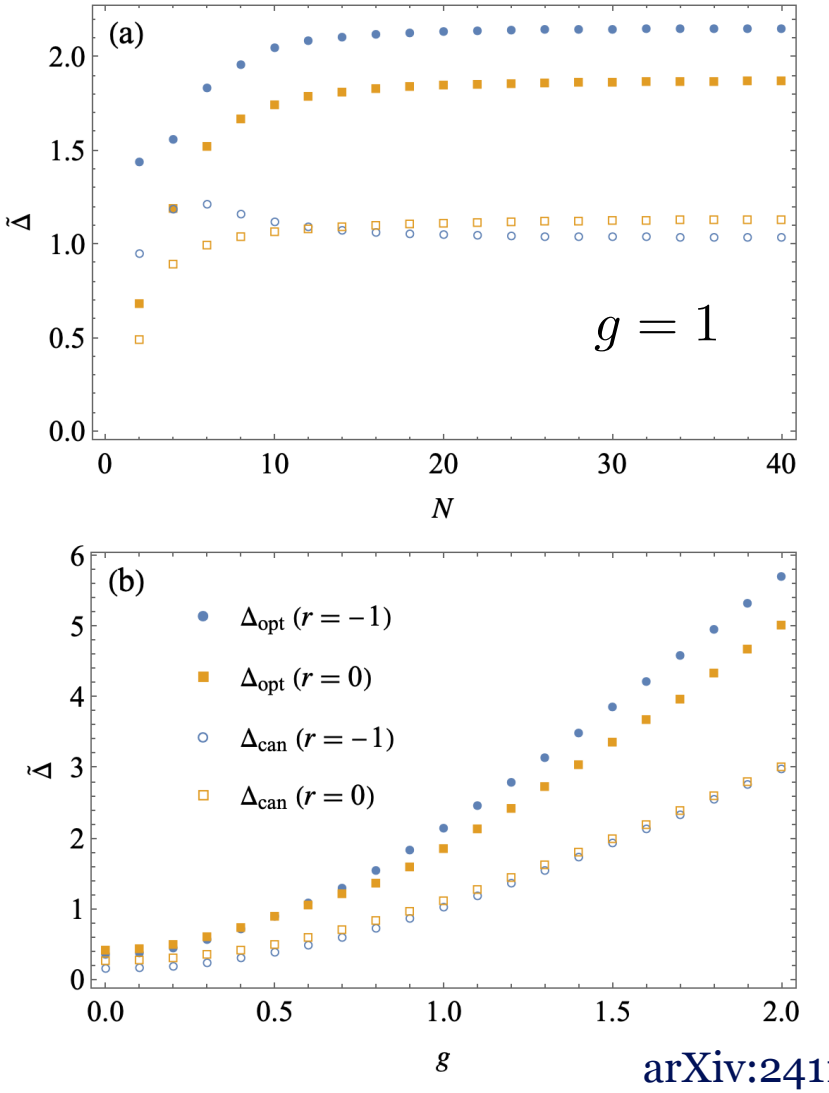
$$\boldsymbol{h} = -J[\frac{2}{N}(\boldsymbol{S}_z^2 + r\boldsymbol{S}_y^2) + 2g\boldsymbol{S}_x - \frac{1+r}{2}]$$

$$\boldsymbol{S}_k = \sum_{i=1}^N \boldsymbol{\sigma}_i^k/2$$

- Infinite-temperature Lindblad operators

$$\tilde{\mathscr{A}} = \{\boldsymbol{S}_x, \boldsymbol{S}_y, \boldsymbol{S}_z\}$$

- Optimize $\tilde{\gamma}_x, \tilde{\gamma}_y$ and $\tilde{\gamma}_z$ to enhance Δ

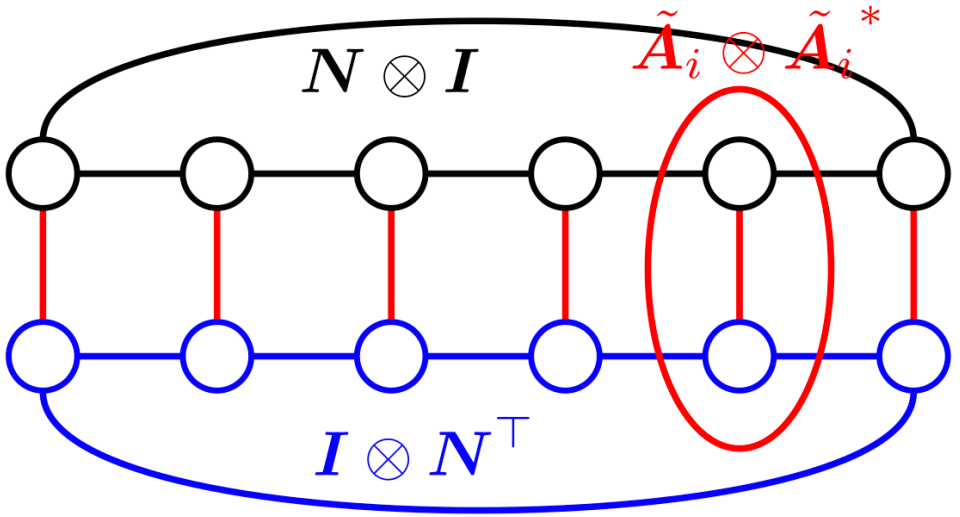


arXiv:2411.03420

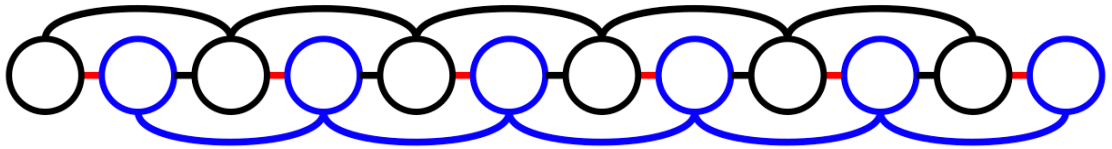
1D Transverse Ising model

$$\mathcal{H} = \sum_{i=1}^{N} \frac{1}{2}(\boldsymbol{N} \otimes \boldsymbol{I} + \boldsymbol{I} \otimes \boldsymbol{N}^{\top}) - \sum_{i=1}^{N} \tilde{\gamma}_i \tilde{\boldsymbol{A}}_i \otimes \tilde{\boldsymbol{A}}_i^{*}$$

- 1D spin ladder



- 2N spin chain



1D Transverse Ising model

- Transverse Ising Hamiltonian

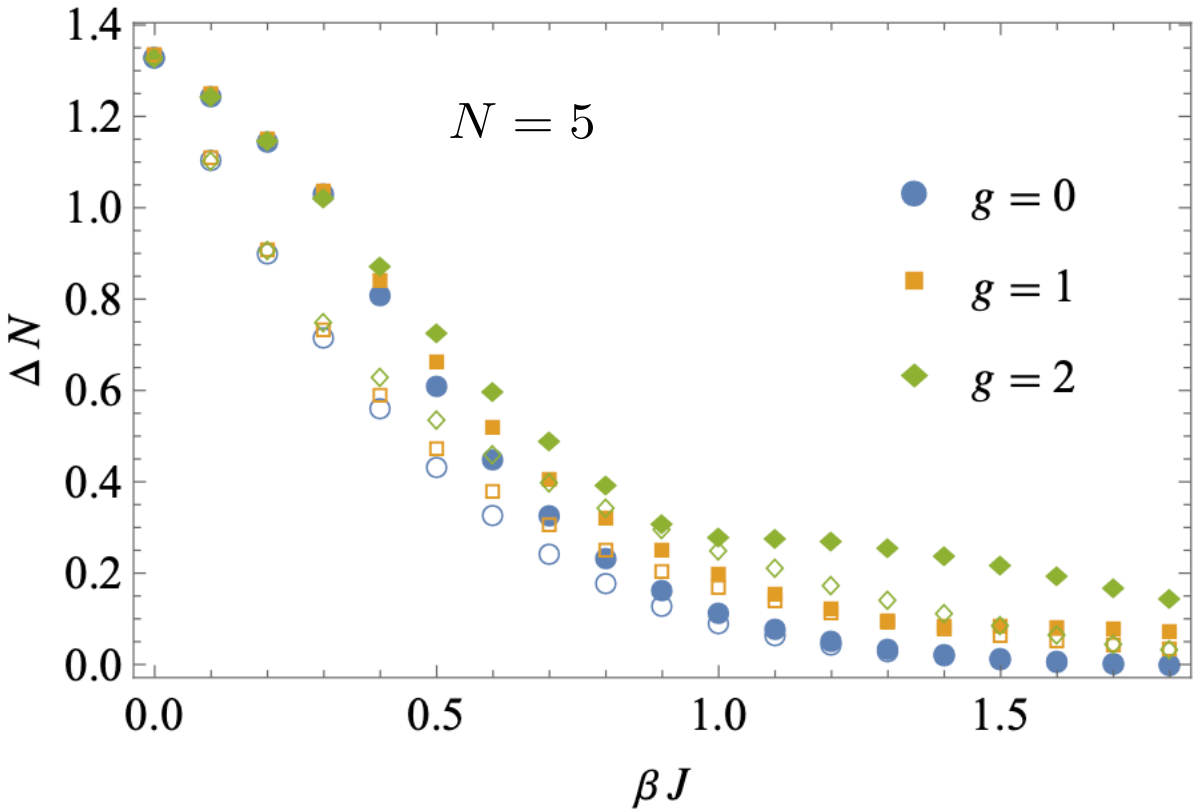
$$\boldsymbol{h} = -J \sum_{i=1}^{N} \left(\boldsymbol{\sigma}_i^z \boldsymbol{\sigma}_{i+1}^z + g \boldsymbol{\sigma}_i^x \right)$$

- Infinite-temperature Lindblad operators

$$\tilde{\mathscr{A}} = \{ \boldsymbol{\sigma}_i^x, \boldsymbol{\sigma}_i^y, \boldsymbol{\sigma}_i^z \}$$

- Optimize $\tilde{\gamma}_x, \tilde{\gamma}_y$ and $\tilde{\gamma}_z$ to enhance Δ

Translational symmetry
Local Z2 symmetry



Filled: optimal kinetic coefficients
Empty: canonical kinetic coefficients

Summary

Quantum Gibbs Sampling

- A family of Lindbladians constructed by dynamical Lie algebra.
- Relevant to quantum thermodynamics, simulation, and algorithms.

Gap optimization

- More jump operators → larger optimal gap
- Symmetry reduces free parameters in $\tilde{\gamma}$
- Gradient-based convex optimization for Δ

Future scope

- Stochastic thermodynamics for these Lindbladians.
- Optimal control of quantum many-body systems.
- MPS Variational and SDP bounds for larger systems.

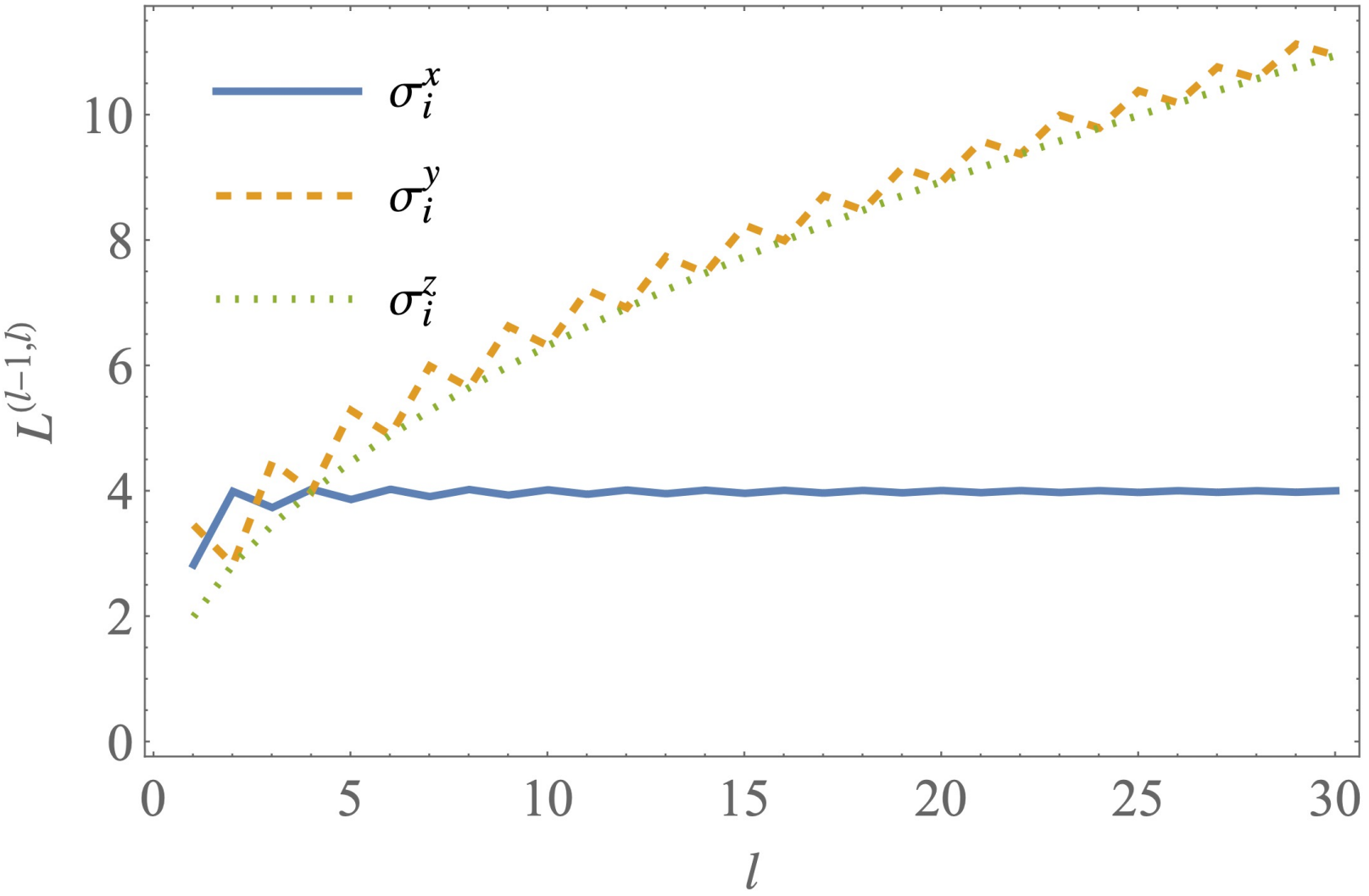


Thank You

arXiv:2411.03420

Universiteit Leiden
The Netherlands

Operator growth in transverse Ising model



$$\tilde{\boldsymbol{A}}_i(t) = e^{i\boldsymbol{h}t}\,\tilde{\boldsymbol{A}}_i e^{-i\boldsymbol{h}t}$$
$$= \mathfrak{a}_i^\dagger e^{L_{\mathfrak{a}_i} t} E_0$$

$$\tilde{\boldsymbol{A}}_i = \boldsymbol{\sigma}_i^{x,y,z}$$

$$N = 20$$