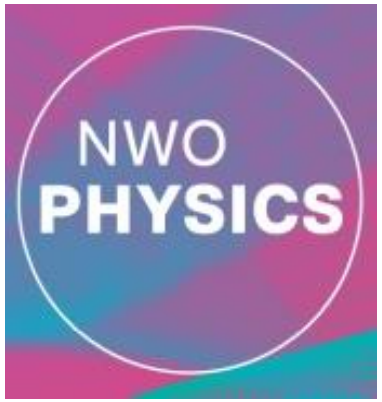
Boosting thermalization of classical and quantum many-body systems

Jinfu Chen | @NWO Physics, 21.1.2025



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arXiv:2411.03420 & ongoing work with **Kshiti Sneh Rai, Patrick Emonts, Donato Farina, Marcin Płodzień, Przemyslaw Grzybowski, Maciej Lewenstein, Jordi Tura**









Motivation

Thermodynamics	Equilibrium	$\rho_\beta \propto e^{-\beta H}$
+		
Equations of motion	Nonequilibrium	$\dot{\rho} = -i[\rho, H] + \cdots$

- Stochastic thermodynamics ➡ Quantum stochastic thermodynamics (?)

Classical detailed balance condition
Continuous-time Markov chain

$$\dot{p} = Lp$$

Quantum detailed balance condition
Lindblad master equation (Lindbladian)

$$\dot{\rho} = \mathcal{L}[\rho]$$

- Preparing classical and quantum thermal states
- Nonequilibrium thermodynamics: thermalization, quench dynamics

U. Seifert, *Rep. Prog. Phys.* **75**, 126001 (2012)
F. Avanzini, M. Bilancioni, V. Cavina, S. Dal Cengio, M. Esposito, G. Falasco, *et al. SciPost Phys. Lect. Notes* 80 (2024).

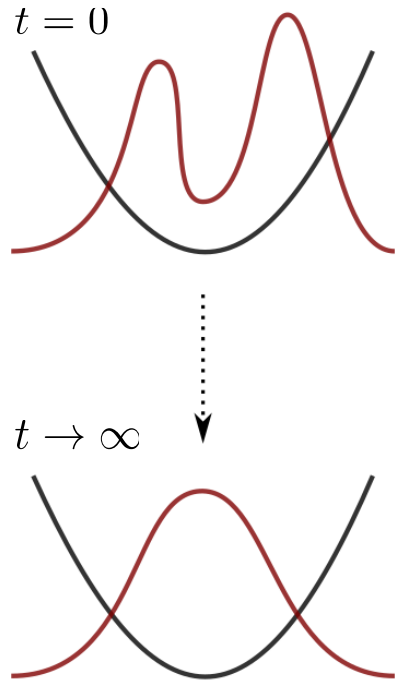
Motivation

- Thermalization dynamics under Lindbladians

$$\dot{\rho} = \mathcal{L}[\rho]$$

$$\rho(t) = \rho_\beta + \rho_1 e^{\lambda_1 t} + \rho_2 e^{\lambda_2 t} + \cdots$$

Slowest relaxing mode



Harmonic potential

- Spectral gap of Lindbladian

$$\Delta = -\mathrm{Re}(\lambda_1) \leq -\mathrm{Re}(\lambda_2) \leq \cdots$$

Maximizing Δ ➡ Fast thermalization ➡ Reducing irreversibility

- For many-body systems, $\mathcal{L}$ has too many free parameters to determine.

Reduced by the symmetry of ρ_β

Motivation

- Quantum algorithm:
- Implement thermodynamic processes on quantum circuits

Nonequilibrium thermodynamic processes → Quantum circuits:
Quantum simulation, quantum computing

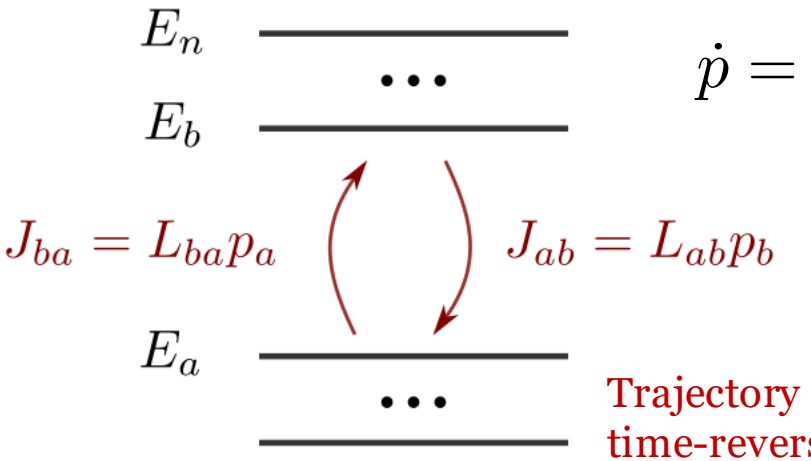
An efficient and exact noncommutative quantum Gibbs sampler,

Chi-Fang Chen, Michael J. Kastoryano, András Gilyén, arXiv:2311,09207

Lindblad engineering for quantum Gibbs state preparation under the eigenstate thermalization hypothesis

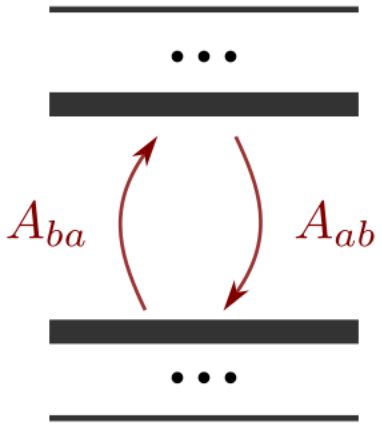
Eric Brunner, Luuk Coopmans, Gabriel Matos, Matthias Rosenkranz, Frederic Sauvage, Yuta Kikuchi, arXiv, 2412.17706

Quantum detailed balance condition



$$\dot{p} = Lp$$

Trajectory in equilibrium:
time-reversal symmetry



Time-reversal symmetry:
A pair of Lindblad operators

$$A_{ba} = \rho_\beta^{1/2} A_{ab}^\dagger \rho_\beta^{-1/2}$$

+

Constraint on coherent dynamics (?)

- Classical detailed balance condition:

$$L_{ab}p_b^{\mathrm{eq}} = L_{ba}p_a^{\mathrm{eq}}$$

- Quantum detailed balance condition:

$$\mathcal{L}^\sharp[\cdot] - \rho_\beta^{-1/2}\mathcal{L}[\rho_\beta^{1/2}\cdot\rho_\beta^{1/2}]\rho_\beta^{-1/2} = 2i[N,\cdot],$$

$$[N,\rho_\beta] = 0$$

F. Fagnola, V. Umanità, *Commun. Math. Phys.* **298**, 523–547 (2010).

Constructing Lindbladians

- Lindblad master equation (Lindbladian)

$$\mathcal{L}[\cdot] = -i[K, \cdot] + \sum_{ln} \gamma_{ln} (A_l \cdot A_n^\dagger - \frac{1}{2}\{A_n^\dagger A_l, \cdot\})$$

- Choose jump operator $\tilde{A}_l$ at infinite temperature

- To prepare $\rho_\beta \propto e^{-\beta H}$, construct jump operator $A_l = \rho_\beta^{1/4} \tilde{A}_l \rho_\beta^{-1/4}$

- Modify the coherent dynamics (see also arXiv:2311,09207)

$$K_{ln} = \frac{i}{2} \sum_{ab} e^{\frac{\beta}{4}(E_a + E_b)} \tanh\left[\frac{\beta}{4}(E_a - E_b)\right] \langle a| \tilde{A}_n^\dagger e^{-\frac{\beta}{2}H} \tilde{A}_l |b\rangle |a\rangle \langle b|$$

arXiv:2411.03420

Constructing Lindbladians

- We rewrite it as

Pairs of transitions

$$\mathcal{L}[\cdot] = \sum_{l,n=1}^{M} \tilde{\gamma}_{ln}(\mathcal{L}_{ln}[\cdot] + \mathcal{L}_{n+M,l+M}[\cdot])$$

$2M$ Lindblad operators

$$\tilde{\boldsymbol{A}} = (\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_M, \tilde{A}_1^\dagger, \tilde{A}_2^\dagger, ..., \tilde{A}_M^\dagger)^T$$

- To prepare $\rho_\beta \propto e^{-\beta H}$, we introduce

$$\mathcal{L}_{ln}[\cdot] = A_l \cdot A_n^\dagger - i[K_{ln}, \cdot] - \frac{1}{2}\{A_n^\dagger A_l, \cdot\}$$

$$A_l = \rho_\beta^{1/4} \tilde{A}_l \rho_\beta^{-1/4}$$

$$K_{ln} = \frac{i}{2} \sum_{ab} e^{\frac{\beta}{4}(E_a + E_b)} \tanh\left[\frac{\beta}{4}(E_a - E_b)\right] \times \langle a| \tilde{A}_n^\dagger e^{-\frac{\beta}{2}H} \tilde{A}_l |b\rangle |a\rangle \langle b|$$

$$A_l \cdot A_n^\dagger + A_{n+M} \cdot A_{l+M}^\dagger$$

$$-i[K_{ln}, \cdot] - \frac{1}{2}\{A_n^\dagger A_l, \cdot\}$$

Satisfy quantum detailed balance condition

Parent Hamiltonian

- Parent Hamiltonian in thermofield double space

$$\mathcal{H}[\cdot] \equiv -\rho_\beta^{-1/4}\mathcal{L}[\rho_\beta^{1/4}\cdot\rho_\beta^{1/4}]\rho_\beta^{-1/4}.$$

- The ground state: the steady state of $\mathcal{L}$;
- Excited states: different relaxation modes.

$$\mathcal{H}[\cdot] = \sum_{l,n=1}^{M} \tilde{\gamma}_{ln}(\mathcal{H}_{ln}[\cdot] + \mathcal{H}_{n+M,l+M}[\cdot]),$$

$$\mathcal{H}_{ln}[\cdot] = \frac{1}{2}\{G_{ln},\cdot\} - \tilde{A}_l\cdot\tilde{A}_n^\dagger,$$

$$G_{ln} = \sum_{a,b}\frac{2\langle a|\tilde{A}_n^\dagger e^{-\frac{\beta}{2}H}\tilde{A}_l|b\rangle}{e^{-\frac{\beta}{2}E_a}+e^{-\frac{\beta}{2}E_b}}|a\rangle\langle b|.$$

Quantum detailed balance:
Frustration-free

Symmetric logarithmic derivative

$$\frac{1}{2}\{G_{ln},\rho_\beta^{1/2}\} = \tilde{A}_n^\dagger\rho_\beta^{1/2}\tilde{A}_l$$

Parent Hamiltonian

Detailed balance condition ➡ Frustration-free Hamiltonians

- Lindbladian

$$\mathcal{L} = \sum_i \mathcal{L}_i$$

- Detailed balance condition ensures

$$\mathcal{L}_i[\rho_\beta] = 0, \ \forall i$$

- ρ_β is the unique steady (equilibrium) state

$$\mathcal{L}[\rho_\beta] = 0$$

- Local Hamiltonian

$$H = \sum_i H_i$$

- Frustration free

$$H_i \left|\psi\right\rangle = 0, \ \forall i$$

- Assume that $H \succcurlyeq 0$, with the ground state energy being,

$$H \left|\psi\right\rangle = 0$$

Optimizing gap

- **Frustration-free Hamiltonian**

$$H = \lambda H_1 + (1-\lambda) H_0$$

- Weyl's inequality

$$\mu_{i+j-1}(H) \geq \lambda \mu_i(H_1) + (1-\lambda)\mu_j(H_0)$$

- From Weyl's inequality ($i = j = 1$), the gap is **concave**

$$\Delta(H) \geq \lambda \Delta(H_1) + (1-\lambda)\Delta(H_0)$$

K. S. Rai, **JFC**, P. Emonts, J. Tura, Spectral Gap Optimization for Enhanced Adiabatic State Preparation, arXiv:2409.15433, **focus session on tensor networks**

Symmetry

Maximize the gap Δ ➡ $G_H = G_\psi$

G_ψ: the symmetry group of $|\psi\rangle$
G_H: the symmetry group of H

- **Concavity:** $G_H \geq G_\psi$

The gap increases when H obeys the symmetry of $|\psi\rangle$.

- **Spontaneous symmetry breaking:** $G_H \leq G_\psi$

Otherwise, for the ground state $|\psi\rangle$, $g_H|\psi\rangle$ is another ground state.

K. S. Rai, **JFC**, P. Emonts, J. Tura, Spectral Gap Optimization for Enhanced Adiabatic State Preparation, arXiv:2409.15433, **focus session on tensor networks**

Classical Liouvillians

Quantum Lindbladian $\mathcal{L}$ under quantum detailed balance condition

Parent Hamiltonian in extended Hilbert space

$$\mathcal{H}[\cdot] \equiv -\rho_\beta^{-1/4}\mathcal{L}[\rho_\beta^{1/4}\cdot\rho_\beta^{1/4}]\rho_\beta^{-1/4}.$$

e.g., Lipkin-Meshkov-Glick model

Classical Liouvillian L under detailed balanced condition

$$L_{ab}p_b^{\mathrm{eq}} = L_{ba}p_a^{\mathrm{eq}}$$

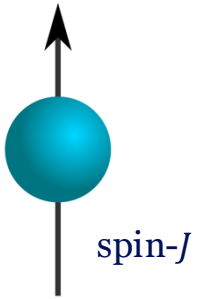
Parent Hamiltonian

$$H = -\rho_\beta^{-1/2}L\rho_\beta^{1/2}$$

e.g., Kinetic Ising models

arXiv:2411.03420

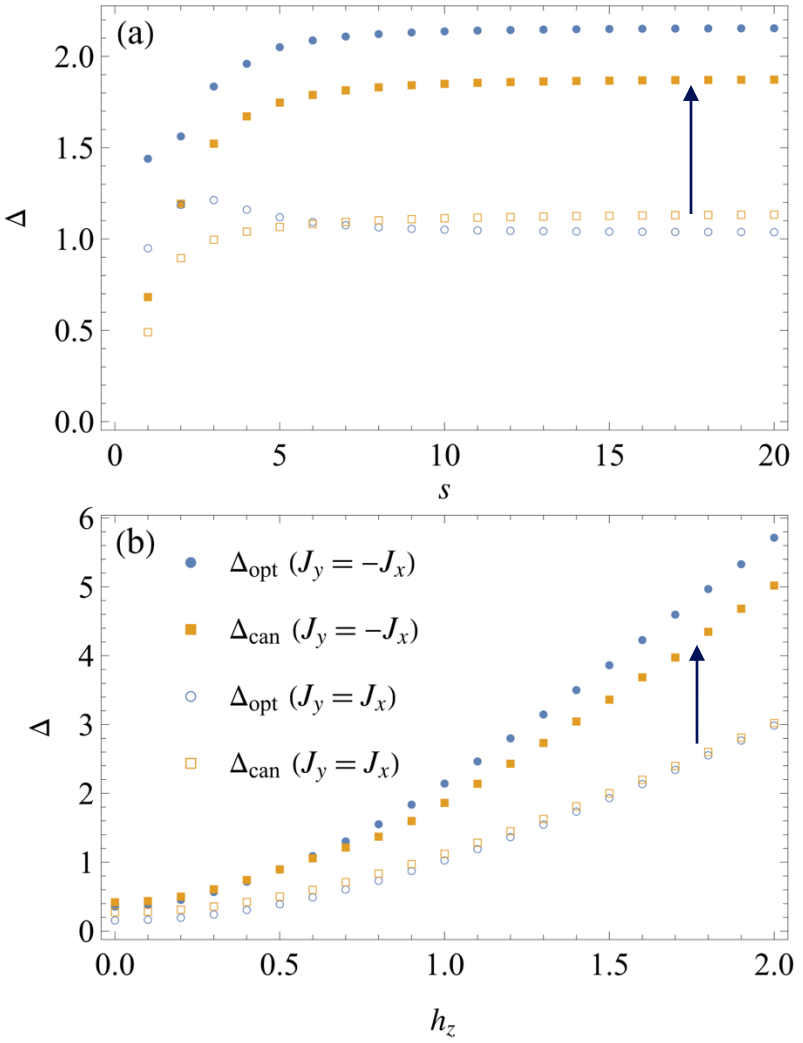
LMG model



$$H = -h_z S_z - \frac{1}{2s}\left(J_x S_x^2 + J_y S_y^2\right)$$

- We construct finite-temperature $\mathcal{L}$ with $A_i = \rho_\beta^{1/4} S_i \rho_\beta^{-1/4}$ and K.

- We optimize $\tilde{\gamma}_{ij}$ (3×3 symmetric matrix) under fixed $\mathrm{Tr}(\mathcal{L})$ to maximize the gap Δ



arXiv:2411.03420

1D Kinetic Ising model model

- **Commuting local terms**

$$H^{\mathrm{cl}} = -\sum_{i=1}^{N} J\sigma_i^z\sigma_{i+1}^z.$$

Translational symmetry
Local Z2 symmetry

- **Single spin flip**



2 kinetic coefficients

$\Gamma,\ \delta$

- **Double spin flip**

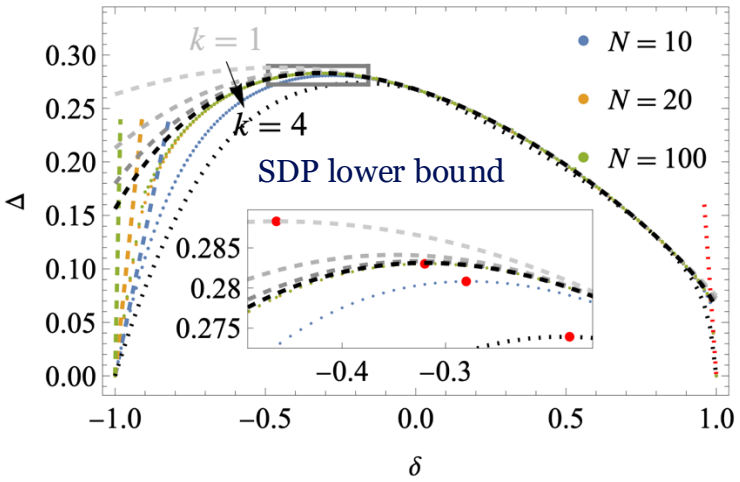


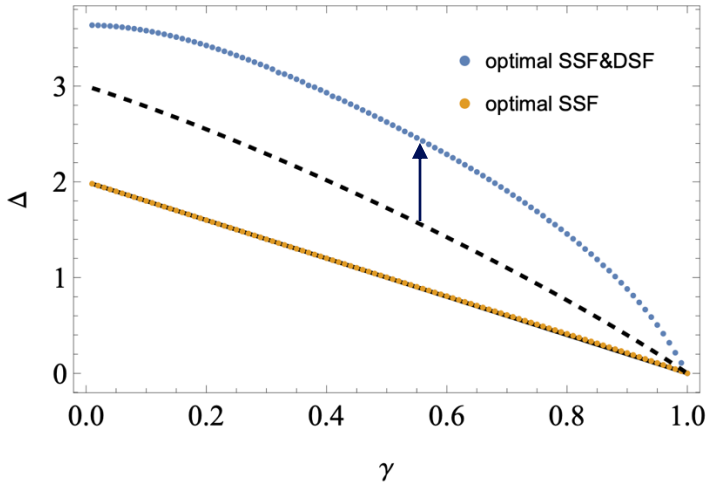
4 kinetic coefficients

$\Gamma,\ \delta_1,\ \delta_2,\ \delta_3$

- **Classical master equation:** $K = 0$
- **We optimize δ under fixed Γ to maximize the gap Δ**

Variational upper bound





arXiv:2411.03420

Thermodynamics

- A family of Lindbladians under quantum detailed balance.
- Relevant to quantum thermodynamics, simulation, and algorithms.

Gap optimization

- Engineering transient dynamics for fast relaxation to thermal states.
- Symmetry reduces free parameters in $\tilde{\gamma}$
- Maximizing Δ.

Future scope

- Establish stochastic thermodynamics for these Lindbladians.
- Optimizing control of open quantum many-body systems.
- Variational and SDP bounds for larger systems.



Thank You

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