Optimizing Brownian heat engine with shortcut strategy

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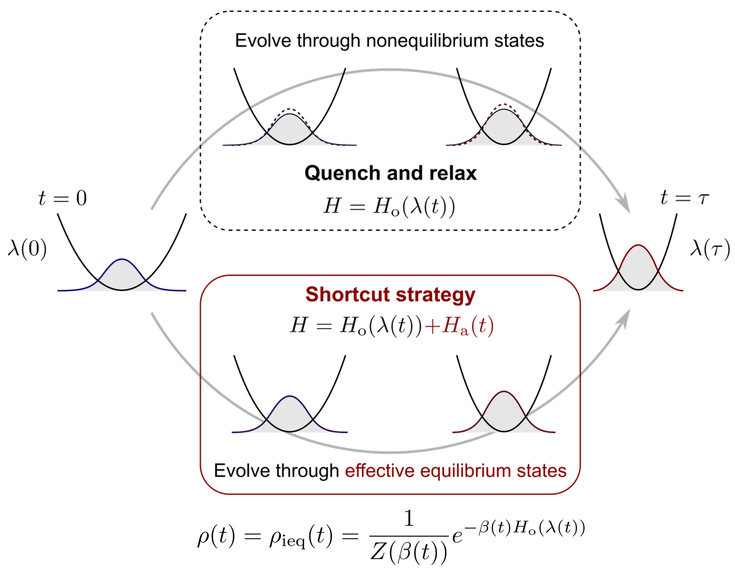
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Abstract

- Shortcut strategy [1] provides a powerful control method to speed up quasistatic thermodynamic processes within finite-time manipulation.
- We use the shortcut strategy to construct and optimize Brownian heat engines in a class of power-law potentials, and formulate a geometric description of energetics with the thermodynamic length.
- A tight bound of output power for shortcut-driven heat engines can be reached with the optimal shortcut control.
- We extend previous results of maximum power and efficiency at maximum power in either underdamped limit or overdamped limit to general damped situation.

Shortcut strategy



$$\rho(t) = \rho_{\rm ieq}(t) = \frac{1}{Z(\beta(t))} e^{-\beta(t) H_o(\lambda(t))}$$

System: a Brownian particle in a harmonic potential

Dynamics: Fokker-Planck equation

$$\frac{\partial \rho}{\partial t} = \mathscr{L}[\rho] + \mathscr{D}[\rho]$$

$$\mathscr{L}[\rho] = -\frac{\partial}{\partial x}\left(\frac{\partial H}{\partial p}\rho\right) + \frac{\partial}{\partial p}\left(\frac{\partial H}{\partial x}\rho\right)$$

$$\mathscr{D}[\rho] = \kappa m \frac{\partial}{\partial p}\left(\frac{\partial H}{\partial p}\rho + \frac{1}{\beta}\frac{\partial \rho}{\partial p}\right)$$

Control of Hamiltonians

$$H_o(x,p,\lambda) = \frac{1}{2m}p^2 + \frac{1}{2}m\lambda^2 x^2$$

$$H_a(x,p,t) = \dot\lambda h_\lambda(x,p,\lambda) + \dot\beta h_\beta(x,p,\lambda)$$

Energetics in shortcut processes

Work: $\dot W \equiv \left\langle \frac{\partial H}{\partial t} \right\rangle = \dot W_o + \dot W_{\rm irr}$ Heat: $\dot Q \equiv \iint H \frac{\partial \rho_{\rm ieq}}{\partial t} dx dp = \dot Q_o + \dot Q_{\rm irr}$

Quasistatic: $\dot W_o = \dot\lambda \left\langle \frac{\partial H_o}{\partial \lambda} \right\rangle$ $\dot Q_o = \iint H_o \frac{\partial \rho_{\rm ieq}}{\partial t} dx dp$

Irreversible: $\dot W_{\rm irr} = \left\langle \frac{\partial H_a}{\partial t} \right\rangle$ $\dot Q_{\rm irr} = \iint H_a \frac{\partial \rho_{\rm ieq}}{\partial t} dx dp$

First law: $\frac{\partial \langle H_o \rangle}{\partial t} = \dot W_o + \dot Q_o$ $\frac{\partial \langle H_a \rangle}{\partial t} = \dot W_{\rm irr} + \dot Q_{\rm irr}$

$$W_{\rm irr} = \langle H_a \rangle(\tau^+) - \langle H_a \rangle(0^-) - Q_{\rm irr}$$

Maximizing output work $-W_o - W_{\rm irr}$ = maximizing $Q_{\rm irr}$

Irreversible heat:

$$Q_{\rm irr} = -\int_0^\tau \begin{pmatrix} \dot\lambda & \dot\beta \end{pmatrix} \mathbf{g} \begin{pmatrix} \dot\lambda \\ \dot\beta \end{pmatrix} dt,$$

Metric:

$$\mathbf{g} = \kappa m \begin{pmatrix} \left\langle \frac{\partial h_\lambda}{\partial p}\frac{\partial h_\lambda}{\partial p} \right\rangle & \left\langle \frac{\partial h_\lambda}{\partial p}\frac{\partial h_\beta}{\partial p} \right\rangle \\ \left\langle \frac{\partial h_\beta}{\partial p}\frac{\partial h_\lambda}{\partial p} \right\rangle & \left\langle \frac{\partial h_\beta}{\partial p}\frac{\partial h_\beta}{\partial p} \right\rangle \end{pmatrix}$$

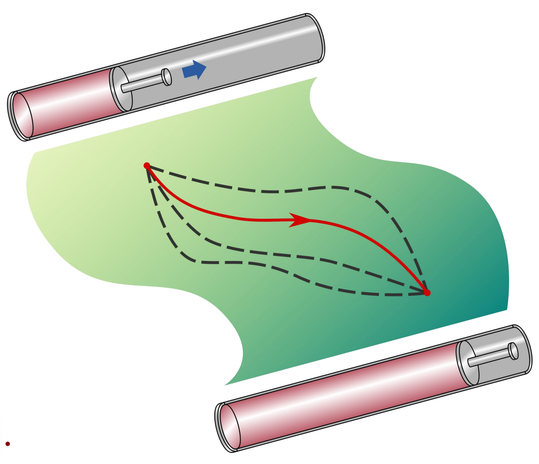
Thermodynamic length:

$$\mathcal{L} \equiv \int_0^1 \sqrt{\begin{pmatrix} \tilde\lambda'(s) & \tilde\beta'(s) \end{pmatrix} \mathbf{g} \begin{pmatrix} \tilde\lambda'(s) \\ \tilde\beta'(s) \end{pmatrix}} ds.$$

Using Cauchy-Schwarz inequality, irreversible work is bounded by

$$W_{\rm irr} = -Q_{\rm irr} \geq \frac{\mathcal{L}^2}{\tau} > 0$$

Equality is reached for the optimal shortcut protocol by varying control parameters (λ and β) with a constant velocity of thermodynamic length [2].

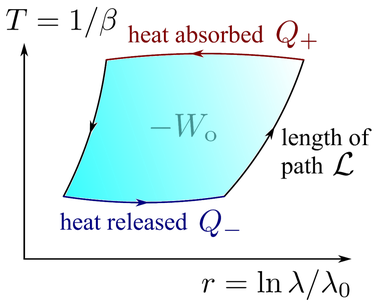


Brownian heat engines with shortcut strategy

Consider a closed path of the cycle in control parameter space.

Output work is bounded by

$$-W = -W_o - W_{\rm irr} \leq -W_o - \mathcal{L}^2/\tau.$$



Power: $P \equiv -W/\tau$

Efficiency: $\eta \equiv \frac{-W}{Q_+} = \frac{-W_o - \mathcal{L}^2/\tau}{-W_o/\eta_o - \mathcal{L}_+\mathcal{L}/\tau}$

To maximize output power, we choose $\tau_{\max} = 2\mathcal{L}^2/(-W_o)$.

Maximum power:

$$P_{\max} = \frac{(-W_o)^2}{4\mathcal{L}^2}$$

Efficiency at maximum power:

$$\eta_{\rm EMP} = \frac{\eta_o}{2 - \eta_o \mathcal{L}_+/\mathcal{L}}$$

η_o : efficiency of quasistatic cycle

Results for harmonic potential

Hamiltonian: $H_o = \frac{1}{2m}p^2 + \frac{1}{2}m\lambda^2 x^2$

Shortcut control [3]: $h_\lambda = \frac{1}{2\kappa}\left[\frac{1}{\lambda m}(p - \kappa m x)^2 + m\lambda x^2\right]$ $h_\beta = \frac{p^2 + (p - \kappa m x)^2 + 2m^2\lambda^2 x^2}{4\kappa\beta m}$

Explicit results of quasistatic work and thermodynamic length:

$$-W_o = -\oint T dr$$ "Euclidean" area of closed path

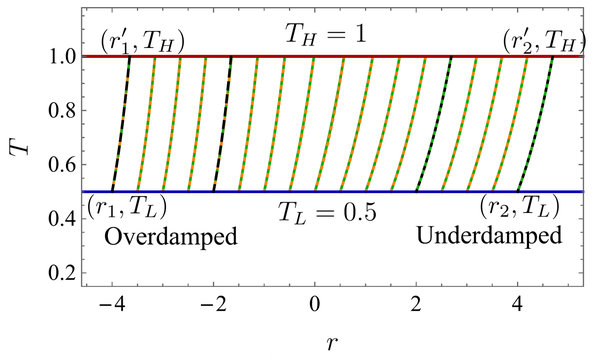
$$\mathcal{L} = \oint \sqrt{\frac{(T dr - dT)^2}{\kappa T} + \frac{\kappa}{(\lambda_0 e^r)^2}\frac{1}{T}\left(T dr - \frac{dT}{2}\right)^2}$$

Dissipation in underdamped limit $\kappa \ll \lambda$ — Dissipation in overdamped limit $\kappa \gg \lambda$

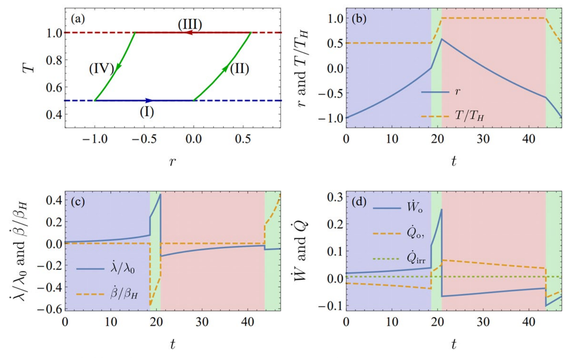
Construction of the heat-engine cycle:

two isothermal processes $T = T_L, T_H$ + two connecting processes $T = T_L e^{\alpha(r - r_0)}$

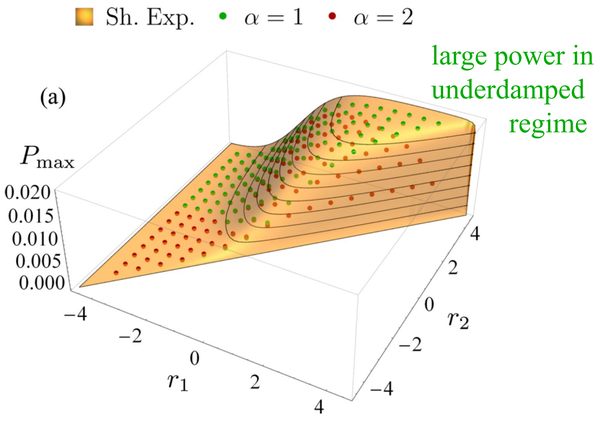
Cycle diagram



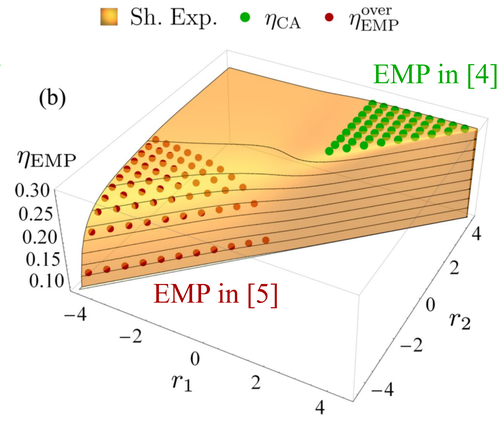
Control of cycle



Maximum power



Efficiency at maximum power



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