Bose-Fermi duality in a quantum Otto heat engine with trapped repulsive bosons

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Abstract

Quantum heat engine with ideal gas has been well studied, yet the role of interaction was seldom explored. We construct a quantum Otto heat engine with N repulsive bosons in a one-dimensional (1D) hard wall box. With the advantage of exact solution using the Bethe ansatz, we obtain not only the exact numerical result of efficiency in all interacting strength c but also analytical results for strong interaction. We find the efficiency η approaches to the one of noninteracting case $\eta_{\rm non} = 1-(L_1/L_2)^2$ for strong interaction with the asymptotic behavior $\eta \sim \eta_{\rm non} - 4(N-1)L_1(L_2-L_1)/(cL_2^3)$. Here L_1 and L_2 are two trap sizes during the cycle. Such a phenomenon reflects the duality between 1D strongly repulsive bosons and free fermions. We observe and explain the appearance of a minimum efficiency at a particular interacting strength and study its dependence on the temperature.

1D Trapped Repulsive Bose Gas

A practical heat engine usually contains many particles as the working medium. The internal interaction of the particles might affects the performance of the heat engine. We design the quantum Otto heat engine with trapped 1D repulsive Bose gas [2] to study the effect of the internal interaction. The Hamiltonian is

$$H(L,c) = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{c}{m}\sum_{i<j}\delta(x_i - x_j) + V(\{x_i\}),$$

where the trap potential $V(\{x_i\})$ is the infinite square potential with the trap size L. The Bethe equation [2] requires the wave vectors to satisfy

$$k_i L = \pi n_i + \sum_{j\neq i}\left(\arctan\frac{c}{k_i-k_j} + \arctan\frac{c}{k_i+k_j}\right).$$

Here, $\{n_i\}$ is a set of ordered positive integers, and gives an eigenstate of the many-body system. The corresponding energy of the eigenstate is

$$E_{\{n_i\}}^{(L,c)} = \frac{1}{2m}\sum_{i=1}^{N} k_i^2 \xrightarrow{cL\gg 1} \frac{\pi^2}{2m}\frac{\sum_{i=1}^{N}(n_i+i-1)^2}{\left[L+\frac{2(N-1)}{c}\right]^2}.$$

the Otto cycle

The Otto cycle consists of four strokes.

Stroke I (1 → 2): Isochoric heating.

$$Q_h = \langle H\rangle_2 - \langle H\rangle_1$$

Stroke II (2 → 3): Adiabatic expansion.

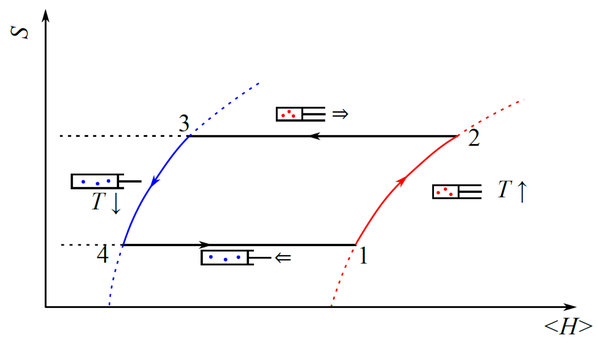
$$W_1 = \langle H\rangle_3 - \langle H\rangle_2$$

Stroke III (3 → 4): Isochoric cooling.

$$Q_c = \langle H\rangle_4 - \langle H\rangle_3$$

Stroke IV (4 → 1): Adiabatic compression.

$$W_2 = \langle H\rangle_1 - \langle H\rangle_4$$



Entropy-energy (S-$\langle H\rangle$) diagram for the quantum Otto heat engine.

We consider quantum adiabatic approximation for the adiabatic processes. The trap size is modulated slowly enough to keep the occupation number $p_{\{n_i\}}^{(j)}$ of different eigenstates unchanged. After a whole cycle, the efficiency of the Otto cycle is determined by

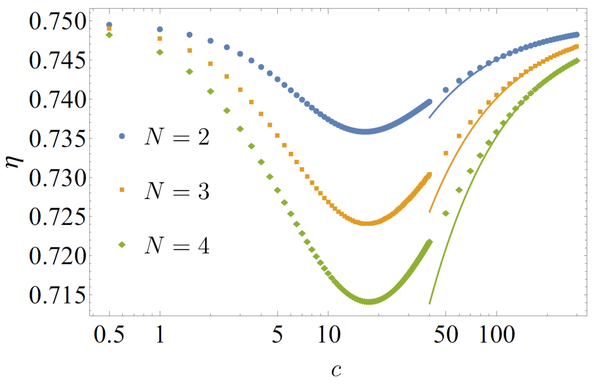
$$\eta = 1 - \frac{|Q_c|}{Q_h} = \frac{\sum_{\{n_i\}}\left[p_{\{n_i\}}^{(2)} - p_{\{n_i\}}^{(4)}\right]\left[E_{\{n_i\}}^{(L_1,c)} - E_{\{n_i\}}^{(L_2,c)}\right]}{\sum_{\{n_i\}}\left[p_{\{n_i\}}^{(2)} - p_{\{n_i\}}^{(4)}\right]E_{\{n_i\}}^{(L_1,c)}}.$$

Efficiency and Interaction

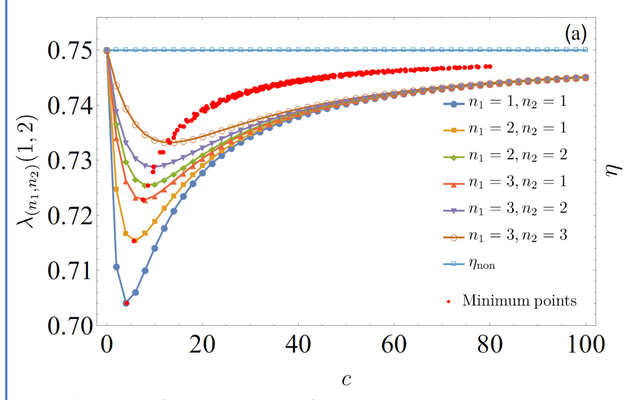
The efficiency at strong interaction limit (the solid curve) approaches to the one of noninteracting bosons or fermions

$$\eta = 1 - \left[\frac{L_1 + \frac{2(N-1)}{c}}{L_2 + \frac{2(N-1)}{c}}\right]^2 \approx \eta_{\rm non} - \frac{4L_1(L_2-L_1)}{L_2^3}\frac{(N-1)}{c}.$$

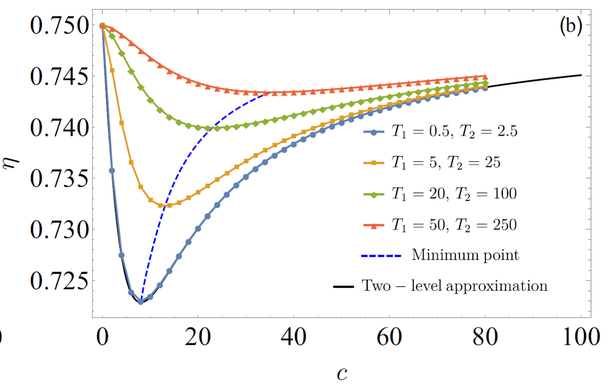
The recovery of the efficiency at strong coupling limit is caused by the duality between fermions and interacting bosons in 1D [3].



The exact numerical result of the efficiency with different interacting strength c. The temperatures are chosen as $T_2 = 50,\ T_4 = 8$ with the trap size $L_1 = 1,\ L_2 = 2$.



Ratio of the energy for two bosons $N = 2$.



Efficiency at different temperatures.

The efficiency is related to the ratio of the energy

$$\eta = \frac{\sum_{\{n_i\}}\left[p_{\{n_i\}}^{(2)} - p_{\{n_i\}}^{(4)}\right]E_{\{n_i\}}^{(L_1,c)}\lambda_{\{n_i\}}(L_1,L_2)}{\sum_{\{n_i\}}\left[p_{\{n_i\}}^{(2)} - p_{\{n_i\}}^{(4)}\right]E_{\{n_i\}}^{(L_1,c)}},$$

Here, $\lambda_{\{n_i\}}(L_1,L_2) = 1 - E_{\{n_i\}}^{(L_2,c)}/E_{\{n_i\}}^{(L_1,c)}$ is the ratio of the energy.

Conclusion

- For weak interaction, the efficiency of the Otto heat engine is lower than the one of noninteracting case. For strong interaction, the efficiency approaches to the one of noninteracting case.
- For fixed interacting strength, the efficiency decreases with the lower temperatures of the reservoirs. The minimum value of efficiency is related to the dip of $\lambda_{\{n_i\}}(L_1,L_2)$.

Reference

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[2] M. Gaudin, Phys. Rev. A **4**, 386 (1971).
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